Complexity of Normal Forms on Structures of Bounded Degree

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Wednesday, 1st of March, 2017, 1:00 p.m., room TEL 716

Abstract

Normal forms express semantic properties of logics by means of syntactical restrictions. For a logic, they are a link between its expressive power and its algorithmic properties. In particular, Gaifman normal form expresses the locality of first-order logic. It serves as an intermediate step in fixed-parameter tractable model-checking algorithms, parameterised by the size of the input formula, for sparse graph classes. However, in most cases, a non-elementary blow-up of the Gaifman normal form is unavoidable and leads to an enormous parameter-dependency.

We consider classes of structures of bounded degree, where this non-elementary blow-up can be avoided, and focus on extensions of first-order logic by unary counting quantifiers. We generalise a local normal form by Hanf and show that formulae permit Hanf normal form only if all quantifiers are ultimately periodic. Furthermore we show that, in this case, Hanf normal form can be computed in elementary time. This leads to elementary algorithms for model-checking, Feferman-Vaught decompositions, and (for the restricted case of first-order logic) Gaifman normal form. For all these algorithms we provide matching lower bounds. In another direction, we use a locality theorem in the manner of Hanf's theorem to show that for formulae that are preserved under extensions (homomorphisms), existential (existential-positive) formulae can be computed in elementary time.