Locality of First-Order Logic and the Expressive Power of Successor-Invariant FO

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It is a well-known fact that first-order logic (FO) cannot express the property that a graph is connected. This can easily be proved either by a compactness argument or by using Ehrenfeucht-Fraisse games. This is just one example showing a general weakness of FO: FO can express only local properties. The famous result of Gaifman formalizes the notion of locality and establishes a corresponding normal form for FO formulas. We review the proof of Gaifman's theorem. If distances between any two elements of a structure are small, Gaifman's theorem is of no interest as in this case the whole model coincides with some local neighborhood. The class of ordered graphs is an example for this. We consider order-invariant FO, where an order is added as an auxiliary relation, but the truth of a sentence must not depend on the particular chosen order. Surprisingly, order-invariant FO can only express local queries as well. Note that this does not imply that we have a Gaifman normal form for FO, as required by efficient model-checking algorithms. Engelmann, Kreutzer and Siebertz recently showed how to efficiently solve the model-checking problem of successor-invariant FO on planar graphs. The question whether successor-invariant FO is stronger than FO on planar graphs remained open. We review an example due to Rossman which separates successor-invariant FO from plain FO on finite relational structures.